A physics-based statistical model for human gait analysis

Petrissa Zell and Bodo Rosenhahn

Institut für Informationsverarbeitung (TNT)
Leibniz Universität Hannover, Germany
{zell,rosenhahn}@tnt.uni-hannover.de

Abstract. Physics-based modeling is a powerful tool for human gait analysis and synthesis. Unfortunately, its application suffers from high computational cost regarding the solution of optimization problems and uncertainty in the choice of a suitable objective energy function and model parametrization. Our approach circumvents these problems by learning model parameters based on a training set of walking sequences. We propose a combined representation of motion parameters and physical parameters to infer missing data without the need for tedious optimization. Both a k-nearest-neighbour approach and asymmetrical principal component analysis are used to deduce ground reaction forces and joint torques directly from an input motion. We evaluate our methods by comparing with an iterative optimization-based method and demonstrate the robustness of our algorithm by reducing the input joint information. With decreasing input information the combined statistical model regression increasingly outperforms the iterative optimization-based method.

1 Introduction

The central endeavour in many biomechanical studies is to determine joint forces and torques, which act at and across a joint, respectively [11, 6, 9]. These forces summarize all active forces effecting a joint, e.g., exerted by tendons, ligaments and neighboring bone segments. The clinical standard to calculate joint torques is through inverse dynamics, based on the measurement of ground reaction forces (GRF) and joint positions by means of force plates and a motion capture (MoCap) system [18]. Despite being frequently used, the results of this approach have to be treated carefully, because various error sources exist which sometimes have non-negligable effects. Especially the length of estimated lever arms is highly sensitive to marker placement uncertainties and the chosen model for body segment parameters [16, 10].

An alternative method for torque estimation is physical modeling of the human body and simulation of dynamical development via forward dynamics. There already exists a variety of physics-based models for human gait with differing complexity. A relatively simple approach is to model body parts by rigid segments that are linked by joints associated with spring torques. These mass-spring models qualify to describe the human walk adequately without the drawback of a high dimensional parameter space. The simulation of movement can be achieved via forward dynamics, i.e. by integrating the equations of motion (EOM) and simultaneously optimizing model parameters to extremize an objective function (often defined as some form of energetic effort). This method has the advantage of directly accessible joint torques, implemented in the EOM, but provokes high computational cost due to the integration. The closer the model gets to reality, i.e. the higher the degree of freedom (DOF) becomes, the larger the computational cost. The iterative minimization of an objective function without prior knowledge of model parameters is referred to as optimization-based method in the following.

Our approach aims to adopt the benefits of physics-based motion analysis while simultaneously avoiding high computational cost by means of machine learning techniques. Methods like principal component analysis for pattern recognition have already been used to analyse and snythesize human motion by Troje et al. in 2002 [14]. We propose a statistical model that combines the physical parameters of a two dimensional mass-spring model based on [3] with corresponding gait characteristics following this approach.

The data driven learning of physical parameters allows us to include style dependent properties of walking into our framework. These properties comprise subject specific preferences to burden some joints more than others, which is an information usually lost when minimizing a general energy function. Simulations were executed on a training set of MoCap data from Troje et al. [14] to estimate a subspace from the physical and motion parameters. This combined representation, termed combined statistical model (CSM) in the following, enables us to directly infer force patterns from motion data without further optimization. Consequently, we achieve a massive reduction of computation time of force and torque estimation compared to optimization-based methods. The computation time of the regression with our CSM lies in the order of seconds. In contrast to that, the optimization-based method we applied for comparison requires computation times of up to several hours. Within the scope of our CSM we propose two different direct regression methods, namely a k-nearest-neighbours (k-NN) approach and asymmetrical principal component analysis (aPCA) [1]. We evaluate our methods by comparing GRFs and joint positions to ground truth data and knee torques to calculations via inverse dynamics.

To summarize, our **contributions** are as follows:

- We introduce a combined statistical model for human motion and corresponding physical parameters.
- The model allows us to estimate missing data in real-time.
- Finally, we analyze different regression methods for force and torque estimation.

2 Related Work

The incorporation of physics-based models into the analysis and synthesis of movement offers the benefit of physical validity. Typical errors like the sliding of

feet on the ground or the simulation of unstable motions can be avoided. Techniques for motion generation and analysis divide into controller-based methods [21, 13, 15] and optimization-based methods [5, 4, 20, 8].
In 1971 Chow and Jacobson introduced an optimization-based approach to simulate human gait [5]. Since then, optimization techniques have been widely used by researchers on the basis of increasingly complex skeletal and muscoskeletal models. Fleet et al. [4] used a 12-segment articulated body model to estimate joint torques and contact dynamics. Their results show consistently estimated torques for walking and running over a wide spread of subjects. The estimated ground reaction forces (GRF) are a good approximation of the ground truth data concerning the mean value, but differ regarding temporal development.
Xiang et al. [20] used a large-scale physical model in order to predict gait patterns. They applied a predictive dynamics approach to approximate joint angles and torques, minimizing the dynamic effort (sum of integrated squared joint torques). Furthermore, GRFs were calculated inversely. The predicted values are in overall similar to experimental data, though calculated GRFs display noticeable difference concerning shape from data available in the literature.
General issues of optimization-based generation of motion with a large degree of freedom (DOF) model are high computational cost and the need for numerous constraints on the model parameter space. Moreover, the minimization of an energy function to optimize walking parameters is a convenient tool for the synthesis of natural looking gaits in general, but often fails to predict subject specific walking styles. Liu et al. [8] adressed this problem by introducing Nonlinear Inverse Optimization to estimate physics-based style parameters from motion capture (MoCap) data. They used learned parameters for the synthesis of new motion in the respective style. Wei et al. [17] combined statistical motion priors with physical constraints in order to generate physically-valid human motion. These last two approaches aim at the generation of physically realistic motion but do not analyze the consistency of simulated force patterns with ground truth data.
In contrast to the existing works, we propose a framework, that encompasses geometrical properties, motion information and physical parameters in a combined statistical model. Relevant advantages of our method towards state-of-the-art methods are robustness in the case of incomplete input information and low computational cost. The combined parametrization enables us to deduce missing data, such as forces or joint trajectories, in real-time.

3 The Physics-based Statistical Model

The generation of a statistical model that combines motion characteristics with a physical representation requires parameter learning on a training set S. For this purpose, MoCap data from Troje et al. [14] was used. The dataset contains walking sequences of 115 male and female subjects with varying weight (from 44.4 kg to 110 kg), height (from 1.52 m to 1.96 m) and age (from 13 years to 59 years).

3.1 Motion Model

Walking can be considered as a time series of postures $\boldsymbol{p}$ and is represented similiarly to [14] as a linear combination of principal component postures with sinusoidal variation of coefficients,

$$\boldsymbol{p}(t) = \boldsymbol{p}_0 + \boldsymbol{p}_1 \sin(\omega t) + \boldsymbol{p}_2 \sin(\omega t + \Phi_2) + \boldsymbol{p}_3 \sin(2\omega t + \Phi_3) + \boldsymbol{p}_4 \sin(2\omega t + \Phi_4)\,. \quad (1)$$

$\boldsymbol{p}_0$ is the mean posture and $(\boldsymbol{p}_1, \boldsymbol{p}_2, \boldsymbol{p}_3, \boldsymbol{p}_4)$ are principal components, called eigenpostures in the following. ω is the fundamental frequency describing the gait and (Φ_2, Φ_3, Φ_4) are phase delays. In this framework, a posture consists of 15 three-dimensional joint positions, resulting in a 45-dimensional vector $\boldsymbol{p}$. The complete motion parametrization is represented by

$$\boldsymbol{u} = [\boldsymbol{p}_0,\, \boldsymbol{p}_1,\, \boldsymbol{p}_2,\, \boldsymbol{p}_3,\, \boldsymbol{p}_4,\, \omega,\, \Phi_2,\, \Phi_3,\, \Phi_4]^T\,. \quad (2)$$

3.2 Physical Model

Our physical gait model is an extension of a two dimensional mass-spring-model of the lower extremeties and the torso by Brubaker et al. [3]. Our modifications are additional body segments (head and arms) with appropriate springs, a toe-off force, and nonlinear force characteristics for a spring that acts on the stance shank. The linear toe-off force $\boldsymbol{F}_{TO}$ is active during a finite timespan Δt_{TO} at the beginning of a gait step (half of a gait cycle) and accelerates the center of mass (COM) of the rear shank. The force is set to

$$\boldsymbol{F}_{TO} = \iota(1 - \frac{t}{\Delta t_{TO}})[-\sin(\phi_{S2} + \alpha), \cos(\phi_{S2} + \alpha)]^T\,, \quad (3)$$

where ι indicates the initial magnitude and α defines the deviation of the force direction from the orientation of the rear shank segment, given by ϕ_{S2}.
Motivated by research on nonlinear spring design [12], we use a nonlinear spring torque that acts on the stance shank to improve the simulation of natural knee flexion and to cover a greater variety of gait patterns. More precisely we set the spring's resting angle $\phi^{(0)}$ to a fourth order polynomial over the x-position of the whole body's COM x_{CoM} resulting in the torque

$$\tau = -\kappa\left(\phi_{S1} - \phi^{(0)}(\boldsymbol{q}, \boldsymbol{\sigma})\right) - d\dot{\phi}_{S1}, \quad (4)$$

$$\phi^{(0)}(\boldsymbol{q}, \boldsymbol{\sigma}) = \phi^{(0)} + \sum_{k=1}^{4} c_k x_{CoM}^k(\boldsymbol{q}, \boldsymbol{\sigma})\,. \quad (5)$$

The parameters κ and d are spring stiffness and attenuation constant, respectively and the angle ϕ_{S1} describes the orientation of the stance shank. The vector $\boldsymbol{q}$ defines the configuration of the model in the form of segment angles and $\boldsymbol{\sigma}$ describes the subject-dependent geometry, i.e. segment lengths. The temporal state of the physical model is given by the pair $(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t))$ and can be determined by integrating a set of equations of motion, resulting in the dynamic state function

$$(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t)) = \boldsymbol{D}(t, \boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \boldsymbol{\theta}, \boldsymbol{\sigma})\,, \tag{6}$$

where $(\boldsymbol{q}_0, \dot{\boldsymbol{q}}_0)$ indicates the initial state and $\boldsymbol{\theta}$ includes all modeled force parameters, i.e. spring and toe-off force parameters.

A detailed analysis of the effects, that these enhancements have on the simulated gait patterns exceeds the scope of this paper and remains for future work. The focus of this publication lies on the inclusion of statistic knowledge to estimate forces and joint torques.

3.3 Combined Representation

To combine the physical properties with the information about a subject's motion, single gait steps taken from the MoCap walking sequences are approximated using the physical model. The approximation process can be divided into two parts: First, subject-specific body parameters and angular dynamics are estimated from MoCap data. Afterwards, effective torques and forces are approximated via model simulation.

In the first step, the distance between two dimensional cartesian model and MoCap joint coordinates $\boldsymbol{r}_{\text{model}}$ and $\boldsymbol{r}_{\text{MoCap}}$, respectively, is minimized by optimizing body segment lengths and angles over a timespan of several steps,

$$(\boldsymbol{q}(t), \boldsymbol{\sigma}) = \underset{\boldsymbol{q}, \boldsymbol{\sigma}}{\arg\min} \left\{ \sum_j \left| \boldsymbol{r}_{\text{MoCap},j}(t) - \boldsymbol{r}_{\text{model},j}(\boldsymbol{q}(t), \boldsymbol{\sigma}) \right|^2 \right\}. \tag{7}$$

We calculate angular velocities and accelerations by means of finite differences and define the consequent states $(\boldsymbol{q}(t), \dot{\boldsymbol{q}}(t))_{\text{targ}}$ as target for the following model simulation. These target states need to be temporally aligned. For this purpose, heel strike times have to be known and are assumed to take place at time points which exhibit a local maximum in step length.

In the second step of the approximation process we search for physical model parameters, that create a motion which has minimal distance to the target motion. In other words, we simulate a step of the model by evaluating function $\boldsymbol{D}$ from Eq. (6) for a set of key times $\{t_k\}_k$ and minimize the sum of squared approximation errors. The times t_k lie within the estimated timespan T_s for single support of the gait step because our physical model does not include a double support phase.

Since our main interest lies in generating realistic force patterns, we also constrain the model simulation to yield GRF values $\boldsymbol{F}_{\text{sim}}$ within the vicinity of ground truth data $\boldsymbol{F}_{\text{true}}$. The values are normalized, i.e. divided by the total body mass M, for comparability. The optimization problem is formulated as follows,

$$\begin{aligned} &(\boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \boldsymbol{\theta}) = \underset{\boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \boldsymbol{\theta}}{\arg\min} \left\{ \sum_k \left| \boldsymbol{D}(t_k, \boldsymbol{q}_0, \dot{\boldsymbol{q}}_0, \boldsymbol{\theta}, \boldsymbol{\sigma}) - (\boldsymbol{q}(t_k), \dot{\boldsymbol{q}}(t_k))_{\text{targ}} \right|^2 \right\}, \\ &\text{s.t.} \quad |\boldsymbol{F}_{\text{sim}}(\boldsymbol{q}(t_k), \dot{\boldsymbol{q}}(t_k), \ddot{\boldsymbol{q}}(t_k), \boldsymbol{\sigma}) - \bar{\boldsymbol{F}}_{\text{true}}(t_k)| \le \eta_k\,, \end{aligned} \tag{8}$$

with thresholds η_k. We calculate the effective normalized GRF via

$$\boldsymbol{F}_{\text{sim}}(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{\sigma}) = \sum_i \frac{m_i}{M} (\boldsymbol{a}_i(\boldsymbol{q}, \dot{\boldsymbol{q}}, \ddot{\boldsymbol{q}}, \boldsymbol{\sigma}) - \boldsymbol{g}) \,, \tag{9}$$

where $\boldsymbol{a}_i$ is the linear acceleration of segment i and $\boldsymbol{g}$ is the gravitational acceleration vector with magnitude $g = 9.81\,\text{m/s}^2$.
The optimization problems in Eq. (7) and (8) are solved by the interior-point algorithm. The resulting physics-based model parameters are

$$\boldsymbol{v} = [\boldsymbol{q}_0,\, \dot{\boldsymbol{q}}_0,\, \boldsymbol{\theta}, M]^T \,, \tag{10}$$

with appended total body mass M which is known from the training set.
Based on the combined parametrization of $\boldsymbol{u}$ and $\boldsymbol{v}$, it is possible to infer joint torques from joint trajectories and vice versa. We do not perform the reverse regression, since joint torques are typically not available as ground truth data, but instead infer joint trajectories from the GRF. For this objective, we define a set of GRF features $\boldsymbol{f}$. The behaviour of $F_x(t)$ is approximately linear. Therefore, we use the slope of $F_x(t)$ as a feature. For $F_y(t)$ we choose the magnitudes at the two maximum points and the minimum point. This results in a four-dimensional feature vector. In the training set, no ground truth data on GRF vectors exists, which is why we learn the GRF parameters $\boldsymbol{f}$ by the use of our simulated values $\boldsymbol{F}_{\text{sim}}$.
Along with the motion representation from Eq. (2) and the physical parameters from Eq. (10) this yields a combined description of walking in form of a 285-dimensional subject specific vector $\boldsymbol{w}_s = [\boldsymbol{u}_s^T, \boldsymbol{v}_s^T, \boldsymbol{f}_s^T]^T$. We obtain a parametrization for the whole training set by writing the vectors $\boldsymbol{w}_s$ into the columns of a matrix $\boldsymbol{W}$:

$$\boldsymbol{W} = \underbrace{\begin{bmatrix} \boldsymbol{u}_1 \ \ldots \ \boldsymbol{u}_{115} \\ \boldsymbol{v}_1 \ \ldots \ \boldsymbol{v}_{115} \\ \boldsymbol{f}_1 \ \ldots \ \boldsymbol{f}_{115} \end{bmatrix}}_{115 \text{ subjects}} \begin{array}{ll} \in \mathbb{R}^{229} & \text{(motion Eq. (2))} \\ \in \mathbb{R}^{52} & \text{(physical model Eq. (10))} \\ \in \mathbb{R}^{4} & \text{(GRF features)} \end{array} \tag{11}$$

4 Missing Data Estimation

4.1 Direct Regression

The combined statistical model encompasses geometrical properties, dynamical behaviour and the physical basis of a walking subject. All of these features contribute to the characteristics of a gait pattern and their mutual dependency can be used to infer missing data from an incomplete parameter set. We apply two different regression methods: k-nearest-neighbour (k-NN) regression and an asymmetrical projection into the principal component space (aPCA), as introduced by [1] for the reconstruction of occluded facial images. Motivated by sparse representation methods [19, 7], our algorithm first performs a classification of the input data concerning predefined motion features, which divide the training set into five pairs of disjoint subclasses. The focus lies on lower body dynamic, e.g. the knee-flexion at different points of the gait cycle. We classify regarding object to class distances of the known part of the parameter set, as suggested by [2]. The intersection of the best matching classes is defined as sample space for the following regression. For the k-NN regression, we set k equal to the number of vectors $\boldsymbol{w}$ covered by this reduced subject set and iteratively reduce k, if the infered joint torque magnitudes surpass a fixed threshold.

4.2 Iterative Optimization

In order to emphasize the advantage of a combined statistical model, we compare the performance of our regression methods to an alternative iterative optimization-based approach, in which we optimize Eq. (7) and (8) to approximate the motion and calculate suitable forces. In the case of incomplete motion input, i.e. incomplete joint trajectories, we augment Eq. (8) to include a penalty function $E(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta})$ in order to account for the missing joint position information. The energy function is based on dynamic effort,

$$E(\boldsymbol{q}, \dot{\boldsymbol{q}}, \boldsymbol{\theta}) = \frac{1}{T} \int_0^T \left(\alpha \boldsymbol{F}_{TO}^2 + \sum_j \beta_j \tau_j^2 \right) dt \,. \tag{12}$$

We empirically set the weights to $\alpha = 0.1$, $\beta_j = 0.001$ for stance leg, spine and neck joints and $\beta_j = 0.0001$ for swing leg and arm joints. This way, a high penalty is placed on the toe-off force and on the stiff joint torques. We refer to this optimization-based method as OPT.

5 Experiments

We compare the performance of the methods for missing data estimation regarding the deviation of estimates from ground truth data. For this purpose, we measured joint trajectories and GRF vectors of three different test subjects. Recording motion and force data was synchronized and done by a Vicon T-series MoCap system and AMTI force plates, respectively. The laboratory setup is depicted in Fig. 2. The force plate system measures magnitude and direction of GRF vectors, which we compare to estimated two dimensional values, resulting from Eq. (9). Furthermore we determine knee extensor and flexor torques of the stance leg via inverse dynamics. The results are compared to simulated model torques τ_{K1}. We use symmetric mean absolute percentage error (SMAPE) as measure for the deviation of estimated magnitudes and first derivatives from ground truth values. The sum of the resulting SMAPE values is used as error measure ϵ. We include first derivatives in this measure in order to increase the weight of shape discrepancies.

5.1 GRF and Knee Torque Estimation

In the first part of the evaluation process our aim is to find the best approximation of the GRF and the stance knee torque given the full motion parametrization $\boldsymbol{u}$ and with an incomplete set of motion parameters, respectively. Starting with missing left hand trajectory, we successively remove the trajectories of the left elbow, ankle and knee, so that at the final stage the entire motion information of limbs on the left-hand side is unknown. The number of missing input joint trajectories is denoted by N. The results for one example subject can be seen in Fig. 1. The depicted estimates are based on complete input information in a) and missing joint information on the full left-hand side in b). Associated SMAPE values ϵ_F and ϵ_τ of GRF and knee torque estimates, as well as computation times t_c, are listed in Table 1.

As expected the best approximation of ground truth data is achieved by the optimization-based method OPT with zero missing input trajectories. In this case the full joint information is used and no additional energy minimization affects the result. As the input information is reduced the CSM methods increasingly outperform OPT. Especially the k-NN approach shows consistently low SMAPE values. The error measure for the estimated knee torque even decreases with increasing N. Which can be explained by the low number of test subjects combined with the inaccuracy of the inverse dynamics calculation of joint torques, meaning that the corresponding errors coincidentally compensate the errors resulting from missing joint information. Consequently the comparison of GRFs has a higher value and should be the decisive measure for the evaluation of a method.

The computation times of the CSM methods are in the order of seconds for k-NN and deci-seconds for aPCA, respectively. In contrast to that, OPT requires computation times of several hours, highly depending on the initialization of the optimization parameters.

Table 1. SMAPE values ϵ_F and ϵ_τ for GRF and knee torque estimates based on the regression methods described in Sect. 4 with related computation times t_c. N indicates the number of missing input joint trajectories.

	k-NN			aPCA			OPT		
N	ϵ_F	ϵ_τ	t_c [s]	ϵ_F	ϵ_τ	t_c [s]	ϵ_F	ϵ_τ	t_c [s]
0	1.504	1.624	2.994	1.726	2.019	**0.881**	**1.483**	**1.420**	8103
1	**1.504**	**1.586**	2.117	1.780	2.052	**0.934**	1.811	1.851	4358
2	**1.496**	**1.582**	2.112	2.005	2.083	**0.913**	1.890	1.793	3096
3	**1.524**	**1.562**	2.583	1.606	2.169	**0.619**	1.843	2.069	3229
4	**1.528**	**1.565**	2.538	1.594	2.072	**0.669**	1.911	2.188	2092

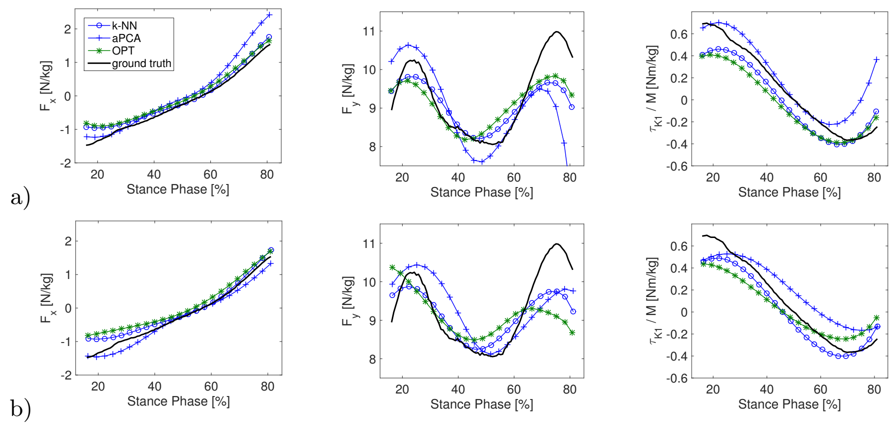

Fig. 1. Comparison between different regression and optimization methods concerning estimated GRF components F_y, F_x and knee torques τ_{K1}. Positive values correspond to flexor torques. The results are based on full joint trajectory information (in a)) and on partial information with $N = 4$ (in b)), respectively. In the case of GRF components, the black line illustrates ground truth data and in the case of joint torques, it represents torques calculated via inverse dynamics. The corresponding evaluation can be found in Table 1.

5.2 Joint Trajectory Estimation

We consider the reverse inference process in a second experiment. Now we want to estimate joint trajectories based on input GRF data. For this experiment we only apply our k-NN algorithm, since it outperformed the other methods in the previous experiment. Furthermore the optimization of joint positions to approximate a target GRF is a highly under-determined problem. Hence, approach OPT would need to be enhanced with multiple constraints on the motion and comparability could not be guaranteed. In addition to the motion feature classification, we reduce the subspace to walkers of matching height to ensure compliant y-positions of the estimated joints.

We deduce the motion vector $\boldsymbol{u}$ from GRF features $\boldsymbol{f}$ and the subjects mass M and height H via k-NN regression of the input vector $[M, H, \boldsymbol{f}]^T$. The results for one example subject are shown in Fig. 3 in form of two-dimensional joint trajectories of the head, the hip, the left knee and the left ankle. Black lines represent ground truth positions and colored lines the estimated values. The mean joint position discrepancy over the time equals 5.6 cm. It is worth mentioning, that the mean position error of the arm joints is 36 % higher than that of the remaining joints. By implication, we can assume that the arm movement has only a minor influence on the GRF. Fig. 4 illustrates the estimated posture of the subject at several time points. Animations of the corresponding motion and the ground truth movement are provided as supplementary material.

Fig. 2. A subject walking across the force plates in our laboratory setup

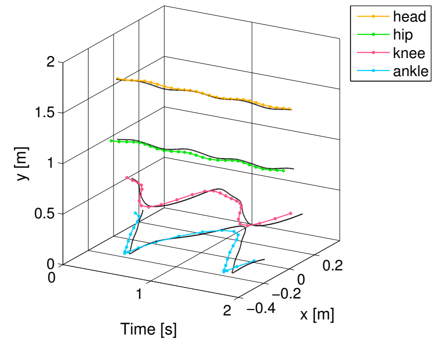

Fig. 3. Two-dimensional joint trajectories. Black lines illustrate ground truth positions and dotted colored lines the corresponding estimates via CSM.

6 Discussion

The experimental results demonstrate the benefit of a combined statistical representation. The method is robust to missing input information and the estimated results approximate the ground truth data as well as our extended physical model allows. The best results for the case of complete input joint information are achieved by means of the iterative optimization-based method OPT, but as soon as we reduce the available information, our k-NN approach outperforms the other methods. Considering the computational effort of the evaluated methods, the CSM causes a significant reduction of computation time from hours to the order of seconds, compared to the iterative optimization-based method. The fact that gradient-descent algorithms generally require a good initial guess of the parameters is not taken into consideration, since we provide our learned parameter space as a set of initial points for the optimization in OPT.
In addition to force estimation, the CSM also enables us to infer a motion from ground reaction force data and subject parameters based on a small set of four features.

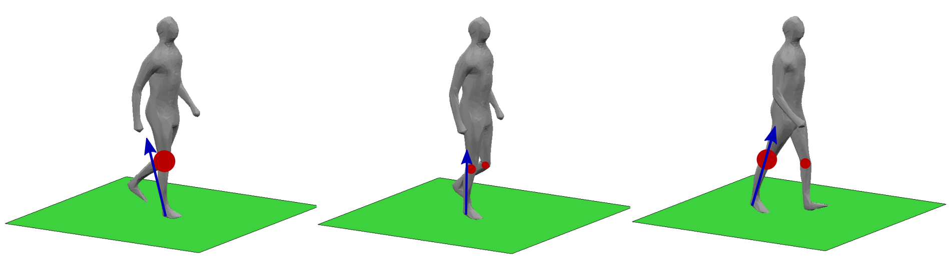

Fig. 4. Frames of the estimated motion based on GRF features $\boldsymbol{f}$. The blue arrow represents the GRF vector and the red discs represent knee joint torques.

References

1. Al-Naser, M., Söderström, U.: Reconstruction of occluded facial images using asymmetrical principal component analysis. Integrated Computer-Aided Engineering 19(3), 273–283 (2012)
2. Boiman, O., Shechtman, E., Irani, M.: In defense of nearest-neighbor based image classification. In: CVPR. IEEE Computer Society
3. Brubaker, M.A., Fleet, D.J.: The kneed walker for human pose tracking. IEEE Conference on Computer Vision and Pattern Recognition (2008)
4. Brubaker, M.A., Sigal, L., Fleet, D.J.: Estimating contact dynamics. In: IEEE 12th International Conference on Computer Vision, ICCV 2009, Kyoto, Japan, September 27 - October 4, 2009. pp. 2389–2396 (2009)
5. Chow, C.K., Jacobson, D.: Studies of human locomotion via optimal programming. Mathematical Biosciences 10(3–4), 239–306 (1971)
6. Fregly, B.J., Reinbolt, J.A., Rooney, K.L., Mitchell, K.H., Chmielewski, T.L.: Design of patient-specific gait modifications for knee osteoarthritis rehabilitation. IEEE Trans Biomed Eng. 54(9), 1687–1695 (2007)
7. Jiang, Z., Lin, Z., Davis, L.S.: Learning a discriminative dictionary for sparse coding via label consistent k-svd. In: Computer Vision and Pattern Recognition (CVPR), 2011 IEEE Conference on. pp. 1697–1704 (2011)
8. Liu, C.K., Hertzmann, A., Popović, Z.: Learning physics-based motion style with nonlinear inverse optimization. ACM Trans. Graph. 24(3), 1071–1081 (2005)
9. Powers, C.M.: The influence of abnormal hip mechanics on knee injury: A biomechanical perspective. JOSPT 40, 42–51 (2010)
10. Rao, G., Amarantini, D., Berton, E., Favier, D.: Influence of body segments' parameters estimation models on inverse dynamics solutions during gait. Journal of Biomechanics 39(8), 1531–1536 (2006)
11. Schmalz, T., Blumentritt, S., Jarasch, R.: Energy expenditure and biomechanical characteristics of lower limb amputee gait: The influence of prosthetic alignment and different prosthetic components. Gait & Posture 16(3), 255–263 (2002)
12. Schmidt, N., Okada, M.: Optimal design of nonlinear springs in robot mechanism: simultaneous design of trajectory and spring force profiles. Advanced Robotics 27(1), 33–46 (2013)
13. Sok, K.W., Kim, M., Lee, J.: Simulating biped behaviours from human motion data. In: Proc. ACM SIGGRAPH '07. p. 107 (2007)
14. Troje, N.F.: Decomposing biological motion: A framework for analysis and synthesis of human gait patterns. Journal of Vision 2(5), 371–387 (2002)
15. Tsai, Y.Y., Lin, W.C., Cheng, K.B., Lee, J., Lee, T.Y.: Real-time physics-based 3d biped character animation using an inverted pendulum model. IEEE Trans. Vis. Comput. Graph. 16(2), 325–337 (2010)
16. Vaughan, C.L.: Are joint torques the holy grail of human gait analysis? Human Movement Science 15(3), 423 – 443 (1996)
17. Wei, X., Min, J., Chai, J.: Physically valid statistical models for human motion generation. ACM Trans. Graph. 30(3), 19:1–19:10 (2011)
18. Whittle, M.W.: Clinical gait analysis: A review. Human Movement Science 15(3), 369–387 (1996)
19. Wright, J., Yang, A.Y., Ganesh, A., Sastry, S.S., Ma, Y.: Robust face recognition via sparse representation. IEEE Trans. Pattern Anal. Mach. Intell. 31(2), 210–227 (2009)

20. Xiang, Y., Arora, J.S., Abdel-Malek, K.: Optimization-based prediction of asymmetric human gait. Journal of Biomechanics 44(6), 683–693 (2011)
21. Yin, K., Loken, K., van de Panne, M.: Simbicon: Simple biped locomotion control. ACM Trans. Graph. 26(3) (2007)