

Spatial Scalable Video Coding using a combined Subband-DCT Approach

Ulrich Benzler

Abstract— A combined subband-DCT approach for spatial scalable video coding is presented. The high resolution input signal is decomposed into four spatial subband signals. The low frequency subband is used as the low resolution signal and is separately coded in the base layer bitstream, the high frequency subband signals are coded in the enhancement layer bitstream. The low resolution signal is reconstructed from the base layer bitstream and the high resolution signal is reconstructed using both the base and the enhancement layer bitstream. Similar to MPEG, DCT-based hybrid coding techniques are applied for the coding of the subband signals, but an improved motion compensated prediction is used for the low resolution signal. Additionally, SNR scalability is introduced to allow a flexible bit allocation for the base and the enhancement layer.

Experimental results at a bitrate of 6 Mbit/s show that the reference coder MPEG-4 Spatial Scalable Profile (SSP) leads to a loss of more than 2.2 dB PSNR compared with non-scalable MPEG-2 coding at the same bitrate, whereas the proposed combined subband-DCT scheme is able to achieve a decrease of less than 0.4 dB in PSNR.

Keywords— Spatial scalability, subband coding, video coding, scalable coding.

I. INTRODUCTION

SPATIAL scalable video coding requires that parts of the encoded data is transmitted in a so-called base layer bitstream and can be decoded separately by a base layer decoder to reconstruct a low resolution signal. Both the base layer and the so-called enhancement layer bitstream are used by the enhancement layer decoder to reconstruct the high resolution signal.

Combined with unequal error protection spatial scalability provides a graceful degradation of the picture quality in the case of transmission errors, which otherwise can result in a complete signal loss. This feature is especially important in mobile communication where channel conditions can degrade significantly over certain periods in time. In combination with temporal and SNR scalability, spatial scalability also enhances multimedia server applications as it provides access to the video at different bit rates in an efficient way, without the need to store multiple single layer bitstreams. This is important when a video server is connected to different users over a heterogeneous network.

In MPEG-2 and MPEG-4 this functionality is provided by the spatial scalable profiles (SSP) [1] [2] using a pyramid coding scheme [3]. The low resolution signal is derived by low pass filtering and subsampling the high resolution input signal. It is encoded separately using a motion-compensated hybrid coder, and the resulting bitstream represents the base layer information of the scalable system.

Ulrich Benzler is with the Institut für Theoretische Nachrichtentechnik und Informationsverarbeitung, University of Hannover, Germany. E-mail: benzler@tnt.uni-hannover.de .

The high resolution signal is also encoded by a motion-compensated hybrid coder, using a second Motion Estimation (ME) with a second set of motion vectors independent from the base layer. In addition, the reconstructed low resolution signal is upsampled and filtered to match the high resolution sampling grid, and is made available as an additional prediction signal for the high resolution signal, see Figure 1. The resulting prediction error of this combined prediction is encoded in the enhancement layer bitstream at the same sampling rate as the original high resolution input signal.

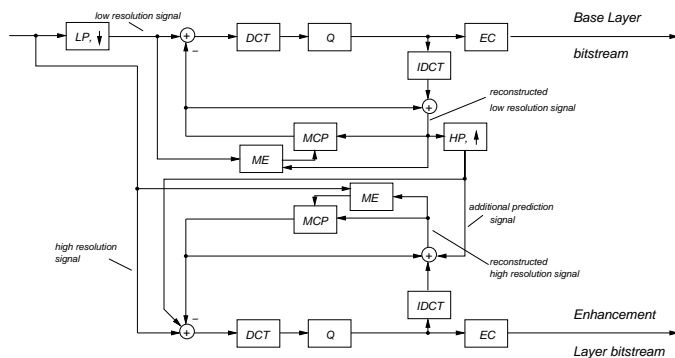


Fig. 1. Block diagram of MPEG SSP pyramid encoder

Therefore, the total number of encoded samples exceeds the number of high resolution input samples by the number of samples encoded for the low resolution signal in the base layer. This disadvantage of an increased sampling rate is not compensated by providing an additional prediction signal for the coding of the high resolution signal.

Additionally, two sets of motion vectors have to be transmitted, which increases the necessary side information.

Furthermore, the independent motion compensated prediction (MCP) of the low resolution signal leads to a loss of coding efficiency, since here the motion compensation is disturbed by aliasing. This aliasing is introduced by the low pass filtering and subsampling in the generation of the low resolution signal.

Thus, an overall coding efficiency can be observed which is not significantly better than “simulcast” where the low resolution and the high resolution signals are coded independently. According to [4] the PSNR for a non-scalable MPEG-2 coding of the ITU-R 601 test sequence “mobile & calendar” at 8 Mbit/s is 30.06 dB. For an MPEG-2 spatial scalable coding using 3 Mbit/s for the SIF base layer and 5 Mbit/s for the enhancement layer the PSNR is 28.44 dB for the ITU-R 601 signal, while independent “simulcast” coding using 5 Mbit/s for the ITU-R 601 signal results in

a PSNR of 27.74 dB.

To overcome the problem of an increased number of encoded samples, the approach used in this paper uses a decomposition of the high resolution input signal into four critically sampled spatial subband signals. The low frequency subband is used as the low resolution signal, which is separately coded and transmitted in the base layer bitstream. This approach is related to the one proposed in [5].

Similar to SSP a separate MCP is used for the low resolution signal, but unlike SSP the MCP for the high resolution signal is performed using both the reconstructed low resolution signal and the high frequency subband signals which are coded in the base and the enhancement layer bitstream, respectively, see Figure 2.

By using the same Motion Estimation (ME) and the same motion vectors in the MCP of both low and high resolution signals the increase in side information can be avoided.

The loss of efficiency due to aliasing in the MCP of the low resolution signal is significantly reduced by the use of a motion and aliasing compensated prediction according to [6].

Rate control is another important element of a coding scheme. In order to match the application demands regarding the bitrate distribution between base and enhancement layer, an SNR scalability for the base layer is proposed.

The material is organized as follows:

Section II describes the structure of the proposed combined subband-DCT scheme, including the additional SNR scalability for the base layer.

Experimental results for the proposed scalable system and for the reference systems are given in Section III.

Finally, conclusions are drawn in Section IV.

II. SPATIAL SCALABLE COMBINED SUBBAND-DCT VIDEO CODING

A. General structure

In the proposed subband-DCT scheme the high resolution input signal is decomposed by a 4-band analysis filter bank AF. This results in one low frequency and three high frequency subband signals, where the low frequency subband signal is used as the low resolution signal, see Figure 2. Similar to SSP, this low resolution signal is encoded separately and transmitted in the base layer bitstream.

But unlike SSP, the subband decomposition reduces the sample rate in the enhancement layer bitstream which now carries only the three high frequency subband signals, each critically sampled at one fourth of the original rate. High resolution images are reconstructed by a synthesis filter bank SF using both the low resolution signal reconstructed from the base layer bitstream and the three high frequency subband signals reconstructed from the enhancement layer bitstream.

The MCP signal for the three high frequency subband signals is generated by applying motion compensated prediction to a previously reconstructed high resolution signal

and subsequent analysis filtering, see Figure 2. The resulting subband prediction signals are subtracted from the high frequency subband input signals, and the prediction error signals are separately DCT transformed, quantized and transmitted in the enhancement layer bitstream.

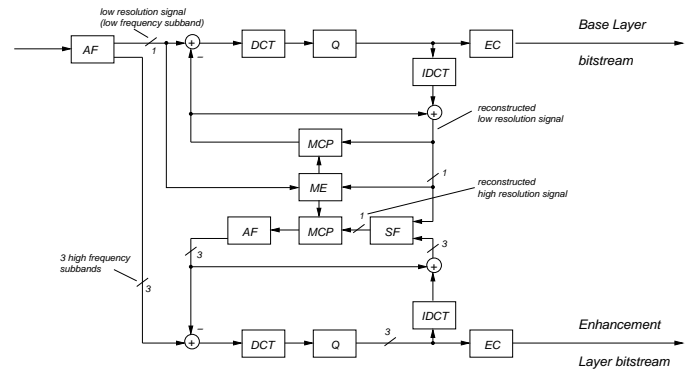


Fig. 2. Block diagram of the combined subband-DCT encoder

The low pass filter used in the analysis filter bank is a symmetrical FIR filter with the coefficients shown in Table I.

It is an adaption of the low pass filter proposed in the MPEG-2 reference encoder TM6 [7] to down-convert ITU-R 601 sequences to the SIF format. This filter is known to provide a good visual quality.

The remaining filters are designed as "generalized quadrature mirror filters" (GQMF) to satisfy the perfect reconstruction condition, meaning that by applying the analysis and the synthesis filter bank subsequently the original signal can be reconstructed perfectly from the subband signals [8]. Their coefficients are also shown in Table I.

TABLE I
FILTER COEFFICIENTS OF THE ANALYSIS AND SYNTHESIS SUBBAND FILTER BANKS.

low pass analysis	high pass analysis	low pass synthesis	high pass synthesis
0	9	9	0
-12	8	-8	12
-10	-18	-18	-10
79	-72	72	-79
142	146	146	142
79	-72	72	-79
-10	-18	-18	-10
-12	8	-8	12
0	9	9	0

Due to the critically sampled subband decomposition, the total number of encoded samples in the proposed scheme is equal to the number of high resolution input samples. Hence, the disadvantage of an increased sample rate is completely avoided.

Furthermore, the MCP for both the low and the high resolution signals uses the same motion vectors, which re-

duces the side information compared with SSP.

To be able to achieve the same coding efficiency as the reference MPEG-2 non-scalable coding scheme, it is necessary to also use half pel accuracy for the MCP of the high resolution signal in the proposed scheme. This corresponds to quarter pel accuracy with respect to the MCP of the low resolution signal, as this signal has half the resolution in both horizontal and vertical direction. But the motion compensation of the low resolution signal is disturbed by aliasing, which is introduced by low pass filtering and sub-sampling in the analysis filter bank. Therefore, a motion and aliasing compensated prediction with quarter pel accuracy has been developed for the low resolution signal, which uses a special Wiener interpolation filter for generating half-pel values (coefficients : $[-8, 24, -48, 160, 160, -48, 24, -8] // 256$). Quarter-pel values are generated by bilinear interpolation between adjacent half and full-pel values. The motion compensation is carried out using overlapping blocks [6]. This motion and aliasing compensated prediction technique has also been proposed for the forthcoming MPEG-4 standard and is included in the first amendment [9].

B. Coding of the spatial subband signals

To keep the implementation of the coding scheme similar to existing coders, for encoding each subband signal a block-based DCT coder similar to MPEG is used. This way the decoder is able to use one single decoding module that can be shared between all subbands.

In order to achieve a high coding efficiency, the properties of the human visual system have to be taken into account for quantizing the subband signals.

Some of the important properties of the human visual system are described by the so-called "Modulation Transfer Function" (MTF) [11]. This function represents the dependence between the spatial frequencies of the input signal and the visibility of distortions. Therefore MPEG-2 uses weighting matrices for the quantization of the different DCT coefficients, which are specially designed to keep the quantization noise for each DCT coefficient below this visibility threshold.

For the use in the combined subband-DCT coder these reference weighting matrices have to be decomposed into sub-matrices, one for each subband signal. A linear interpolation from the 8x8 reference matrices to 16x16 matrices and subsequent division into four 8x8 matrices for the subband signals is not possible, because the analysis filters used in the subband decomposition are subject to aliasing. This aliasing cannot be avoided as filters with a short impulse response have to be used in order to achieve an appropriate subjective image quality for the low resolution signal. Filters with a longer impulse response, which tend to cause less aliasing, would lead to annoying "ringing artifacts" in the low resolution signal.

As the high frequency subband coefficients have a smaller dynamic range than the low resolution coefficients, they are amplified in the analysis filter bank to make them suitable

for the same DCT and entropy coder that is used for the low resolution signal. This amplification by a factor of 2 for the high-low and the low-high subband and by a factor of 4 for the high-high subband is reversed in the synthesis filter bank, so that perfect reconstruction is maintained. This amplification has to be considered in the design of the subband weighting matrices.

For determining the coefficients of the individual subband weighting matrices a method has been developed which is explained in Figure 3. As input signal to the experimental set either the original high resolution signal (for I-frames) or the resulting prediction error after MCP of the high resolution signal (for P- and B-frames) is used. The method consists of a DCT transform of the high resolution input signal and weighted quantization using the corresponding reference 8x8 MPEG-2 quantization matrix. After a subsequent inverse DCT transform the quantized input is available at point "A" in Figure 3. This signal is decomposed into subband signals by the analysis filter bank AF. The subband signals are separately DCT-transformed resulting in four sets of 8x8 reference subband DCT coefficients at point "B" in Figure 3.

In the second branch the high resolution input signal is directly decomposed, and the resulting subband signals are DCT transformed. Each of the resulting four 8x8 input subband DCT coefficients at point "C" in Figure 3 is quantized, and each quantizer is adjusted so that the same distortion as in the corresponding reference DCT coefficient is maintained at point "D" in Figure 3. This process is carried out over a set of test sequences. The resulting mean values are used as the elements for the subband quantization matrices.

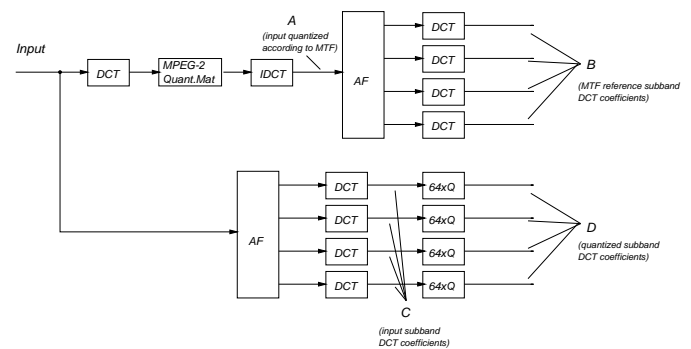


Fig. 3. Experimental set for determining subband weighting matrices

The derived matrices are shown in the Appendix. Note that the amplification of the subband signals is already included.

C. Control of bitrate distribution between base and enhancement layer bitstream

Rate control in the MPEG-2 TM6 reference encoder is achieved by varying the quantizer scale for the DCT coefficients. This is done similarly in the proposed combined subband-DCT scheme. But since here the synthesis of the reconstructed low resolution signal and the three high frequency subband signals together determine the quality of

the reconstructed high resolution signal, the control of the quantizer scales for the low resolution and the high frequency subband signals has to be implemented in a combined way, using a single quantizer scale that is applied to all weighting matrices equally. The bitrate distribution between base and enhancement layer bitstream cannot be controlled by using different quantizer scales.

An analysis of the bit allocation between the base and the enhancement layer bitstream of the proposed coding system shows that the low resolution signal coded in the base layer bitstream needs approx. 75% of the total bitrate.

Similar results can be observed by an analysis of the non-scalable MPEG-2 encoder. Depending on the test sequence 65 to 80% of the overall bitrate is used for coding the spatial frequencies equivalent to those carried in the low resolution in the base layer bitstream of the scalable system. This is due to the low pass characteristic of the MTF of the human visual system. The MTF allows a larger quantization error at high spatial frequencies, leading to a larger number of zero-amplitude coefficients which reduces the demand in bitrate for the high frequency subband signals encoded in the enhancement layer bitstream [11].

In addition the base layer bitstream of the proposed coding system also carries the common side information for motion vectors, prediction information etc., which increases the base layer bitrate even more.

The amount of 75 % base layer bitrate is very unfavourable in most scalable applications. In order to allow a flexible bit allocation to the base and the enhancement layer bitstream, parts of the low frequency subband need to be transmitted in the enhancement layer bitstream. Therefore a so-called "SNR scalability" is applied where a coarsely quantized low frequency subband signal is transmitted in the base layer bitstream, using only 50% of the total bitrate. Additional data for fine quantization of the low frequency subband signal is transmitted in the enhancement layer bitstream, together with the data for the three high frequency subband signals.

This SNR scalability is not needed for the SSP encoder, because it can use an arbitrarily coarse quantized base layer. The reconstructed low resolution pictures are not directly used for the high resolution pictures, as is the case for the proposed subband scheme.

The modified block diagram of the combined subband-DCT encoder with SNR scalable base layer is shown in Figure 4.

In this work "embedded quantizers" [12] are used for SNR scalability of the low frequency subband signal. Embedded quantization means that the signal is subsequently quantized by multiple quantizers which bisect the quantization interval in every stage. In combination with bit plane coding [13] of the DCT coefficients, embedded quantization is able to achieve a coding efficiency that is close to a non-SNR-scalable scheme.

III. EXPERIMENTAL RESULTS

A scalable coding scheme has two performance bounds with respect to the reconstruction error Peak Signal to

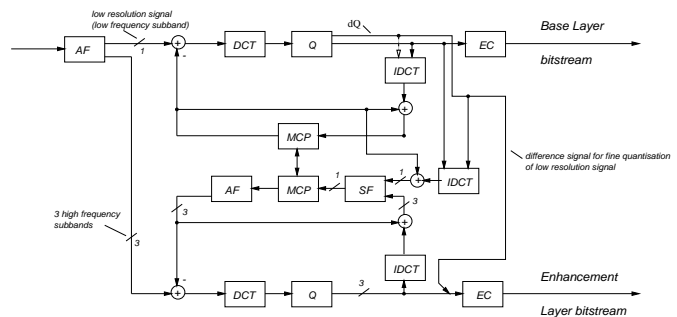


Fig. 4. Block diagram of the combined subband-DCT encoder with SNR scalable base layer

Noise Ratio (PSNR) to compare with :

- The lower bound is the PSNR of the "simulcast" case, where the total bitrate is divided to independently encode the low and the high resolution signal
- The upper bound is the PSNR of the non-scalable coding, that uses the total bitrate only for encoding the high resolution signal

Both the developed scalable subband-DCT coding scheme and the MPEG-4 Spatial Scalable Profile (SSP) are compared with non-scalable MPEG-2 coding and MPEG-2 simulcast at a total bitrate of 6 Mbit/s. The base and the enhancement layer bitrates are 3 Mbit/s each.

The use of interlaced sequences in a multi-resolution coding scheme is very difficult, as the sampling lattice of the low resolution interlaced signal would not be a subset of the high resolution interlaced signal, even if the number of lines is exactly the double [14]. To overcome this problem, a motion compensated de-interlacing of the input video sequence according to [15] is applied. The high resolution signal used in the experiments is a 50 Hz progressive sequence with a spatial resolution of 704x576 pixel, which leads to a low resolution signal of 50 Hz progressive SIF (352x288). Two de-interlaced ITU-R 601 test sequences, Basketball and Bus, are used, and the results for the high resolution are shown in Figure 5.

Since the simulcast and SSP base layer encoders are identical to the non-scalable encoder for the low resolution signal, they produce the same bitstream at 3 Mbit/s. For the scalable subband-DCT the base layer shows a performance which is comparable with non-scalable coding of the low resolution at 3 Mbit/s. The PSNR values are only 0.05 to 0.5 dB below those of the non-scalable MPEG-2 coder. This small decrease is due to the SNR scalability used in the scalable subband-DCT base layer.

But for the high resolution signal the MPEG-4 SSP coding leads to a loss of more than 2.2 dB in PSNR compared with non-scalable coding, which is not significantly better than the results for simulcast coding, see Figure 5. This degradation in PSNR is clearly visible in the reconstructed video sequences.

In contrast to this the PSNR values of the scalable subband-DCT scheme are only 0.4 dB below those of the non-scalable MPEG-2 coding, meaning that nearly no loss in coding efficiency and subjective quality is observed for

the additional scalable functionality.

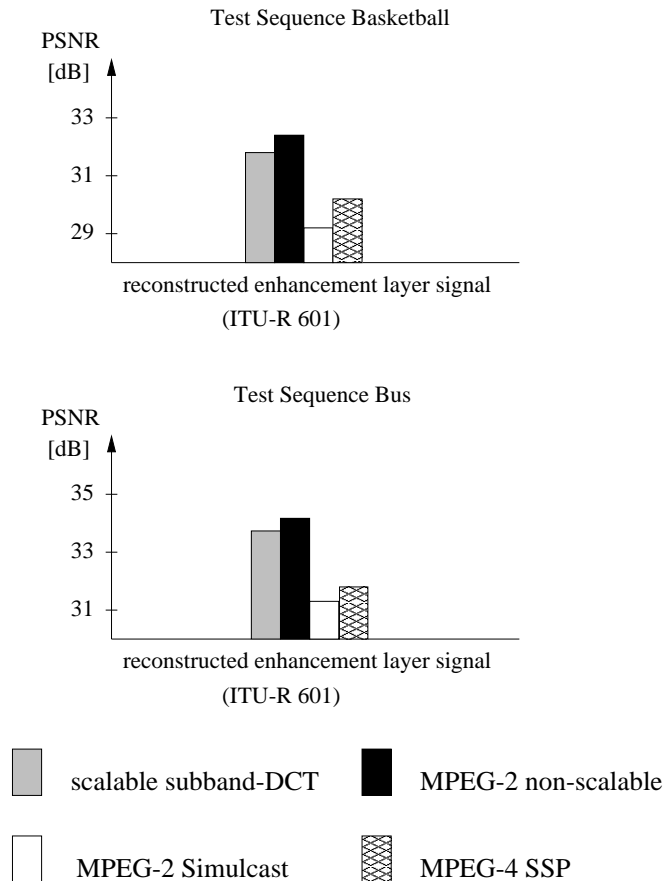


Fig. 5. Simulation results for scalable subband-DCT coding compared with non-scalable MPEG-2 coding, MPEG-2 simulcast and MPEG-4 SSP. Total bitrate 6 Mbit/s, base layer 3 Mbit/s, enhancement layer 3 Mbit/s

IV. CONCLUSIONS

A spatial scalable video coding scheme using a combined subband-DCT approach is presented. It avoids the problem of an increased number of encoded samples, which is observed in the standardized spatial scalable coding scheme (SSP) used in MPEG-2 and MPEG-4, by applying a critically sampled 4-band subband decomposition. The subband filter bank uses filters with short impulse responses to provide an appropriate subjective image quality for the low resolution signal.

In order to use the same motion vectors for both the low and high resolution MCP, a motion and aliasing compensated prediction with quarter pel accuracy is used for the low resolution signal. This technique is also able to reduce the loss of efficiency due to the aliasing disturbed MCP for the low resolution signal. It is included in the forthcoming MPEG-4 standard.

To exploit the properties of the human visual system by quantizer weighting matrices in the same way as MPEG-2 coding, a method for generating weighting matrices for the quantization of the subband DCT coefficients is developed.

In order to allow a flexible bitrate distribution between

the layers, an additional SNR scalability for the base layer is introduced. It allows to transmit parts of the low frequency subband signal in the enhancement layer bitstream, together with the high frequency subband signals. By using embedded quantizers in combination with bitplane coding of the DCT coefficients this can be achieved with nearly no loss in coding efficiency.

Experimental results show that the standardized MPEG-4 SSP coding leads to a decrease of more than 2.2 dB in PSNR compared with non-scalable MPEG-2 coding, which is not significantly better than "simulcast" coding.

In contrast to this the results for the proposed scalable subband-DCT coding show less than 0.4 dB decrease in PSNR when compared with non-scalable MPEG-2 coding. This means that the additional functionality of scalability is provided with nearly no loss in coding efficiency.

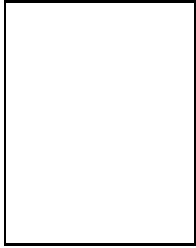
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Ulrich Benzler received his Dipl.-Ing. degree from University of Hannover in 1993 and has been pursuing his PhD at the Institut für Theoretische Nachrichtentechnik und Informationsverarbeitung, University of Hannover, since then. His research interests include image processing, video coding and visual communications. He contributed to the development of the MPEG-4 standard for the visual part, where he chaired the AdHoc groups for the MPEG-4 video verification tests and the visual software integration.

APPENDIX

Weighting matrices for the reference encoder MPEG-2 TM6 and the corresponding developed matrices for quantization of the subband-DCT coefficients. Note that the amplification by a factor of 2 for the high-low subband (upper right part) and low-high subband (lower left part), and by a factor of 4 for the high-high subband (lower right part) is included.

Original MPEG-2 TM6 Intra matrix :

8	16	19	22	26	27	29	34
16	16	22	24	27	29	34	37
19	22	26	27	29	34	34	38
22	22	26	27	29	34	37	40
22	26	27	29	32	35	40	48
26	27	29	32	35	40	48	58
26	27	29	34	38	46	56	69
27	29	35	38	46	56	69	83

resulting Intra subband matrices :

5	5	8	8	9	9	10	10	⋮	24	24	23	23	25	26	27	24
5	6	8	9	10	10	10	10	⋮	24	23	22	22	24	25	26	30
8	8	8	9	11	10	11	11	⋮	26	25	24	24	26	28	28	33
8	8	9	10	11	11	11	11	⋮	26	25	24	24	26	27	27	32
9	9	11	11	12	12	11	12	⋮	28	26	26	25	26	28	27	34
9	9	10	11	12	11	11	11	⋮	27	25	24	25	25	28	26	35
9	10	10	10	11	11	11	11	⋮	26	24	23	24	25	28	26	48
9	9	10	10	11	11	11	12	⋮	26	25	24	25	29	40	39	48
22	22	25	25	26	25	25	26	⋮	58	55	53	55	58	68	56	96
21	21	23	23	24	24	23	24	⋮	55	52	49	50	54	62	53	96
20	20	22	22	22	22	22	24	⋮	53	49	50	53	56	72	57	96
21	20	21	21	22	22	22	25	⋮	54	51	53	62	79	96	96	96
20	19	19	20	21	22	23	27	⋮	55	54	59	75	96	96	96	96
23	23	24	24	25	26	27	40	⋮	62	64	73	96	96	96	96	96
20	19	20	20	21	22	25	35	⋮	59	63	70	96	96	96	96	96
18	23	24	24	25	28	33	48	⋮	73	80	96	96	96	96	96	96

Original MPEG-2 TM6 non-Intra matrix :

16	17	18	19	20	21	22	23
17	18	19	20	21	22	23	24
18	19	20	21	22	23	24	25
19	20	21	22	23	24	25	26
20	21	22	23	25	26	27	28
21	22	23	24	26	27	28	30
22	23	24	26	27	28	30	31
23	24	25	27	28	30	31	33

resulting non-Intra subband matrices :

8	8	8	8	9	9	8	8	⋮	20	20	19	19	20	20	20	25
8	8	8	8	9	9	8	8	⋮	20	19	19	18	19	19	20	24
8	8	9	9	9	9	9	8	⋮	21	20	20	20	21	21	22	28
9	8	9	9	9	9	9	9	⋮	22	20	20	20	20	21	22	33
9	9	9	9	9	10	9	9	⋮	22	21	21	20	21	22	23	47
9	9	9	9	9	9	9	9	⋮	22	21	21	20	20	22	23	48
8	8	9	9	9	9	9	8	⋮	22	20	20	20	21	22	23	48
8	8	9	9	9	9	9	8	⋮	22	20	20	20	21	24	27	48
.....																
20	20	22	22	22	22	22	22	⋮	52	50	49	49	50	55	57	96
19	19	20	21	21	20	20	20	⋮	50	47	45	44	47	50	56	96
18	18	20	20	20	20	19	19	⋮	48	44	43	43	44	53	55	96
18	18	19	19	19	19	19	19	⋮	47	44	44	45	45	60	85	96
18	18	18	19	20	20	19	20	⋮	49	45	44	45	47	56	71	96
20	20	22	22	23	23	22	22	⋮	53	49	49	49	52	61	89	96
19	19	20	21	21	21	21	21	⋮	51	48	47	48	50	53	63	96
22	23	23	24	24	24	22	24	⋮	58	53	53	54	56	58	70	96